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COMMON FIXED POINT THEOREM OF FOUR MAPPING

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ABSTRACT

Srivastava [12], [9], Dubey & Dubey [3], Bhola & Sharma [1] Pandey &Dubey [6]; Considered three self mappings and obtained a unique common fixed point. Here we generalized the contraction used by above authors for five maps and obtained a unique fixed point.

KEYWORDS: Fixed Point Theorems, Mappings, Contraction Mappings, Continuous Mappings, Complete Metric Space

INTRODUCTION

Srivastava [9] used the inequality as follows

[1.1] $$d(E_x, f_y) \le C_1\left[\frac{d(T_x,E_x)d(T_y,F_y)}{d(T_x,T_y)}\right] + C_2\left[\frac{d(T_x,F_y)d(T_y,F_y)}{d(T_x,F_y)}\right]$$

$$+ C_3\left[d(T_x,F_y) + d(T_y,F_y)\right] + C_4\left[d(T_x,F_y) + d(T_y,E_x)\right]$$

$$+ C_5\left[d(T_x,T_y) + d(E_x,F_y)\right] + C_6 d(T_x,T_y);\ \forall\, x,y \in X$$

$$T_x \ne T_y$$

Then, E, F, T have a unique common fixed point if

$$C_i \ge 0;\ \ 0 \le \mathrm{C}_1 + 2(C_2 + C_3 + C_4 + C_5 + C_6) \le 1,$$

$$0 \le 2(C_4 + C_5 + C_6) \le 1;$$

Dubey &Dubey [3] have proved a fixed point theorem for three maps S, T &I of a complete metric space (X, d) satisfying:

[1.2] $$d(S_x,T_y) \le \frac{q\left\{\alpha d(I_x,S_x)d(T_x,T_y) + \beta d(T_y,S_x)d(I_y,T_y) + \gamma d(I_x,T_y)\right\}^2}{\alpha d(T_x,T_y) + \beta d(I_y,S_x) + \gamma d(I_x,T_y)}$$

$$\forall\ \ \alpha \mathrm{d}(\mathrm{T_x},T_y) + \beta d(I_y,S_x) + \gamma d(I_x,I_y) \ne 0$$

$$0 \le q < 1,\ \alpha,\beta,\gamma \ge 0$$

Pandey & Dubey [6] obtained a unique common fixed point of three maps E, F&T satisfying.

[1.3] $d(E_x,F_y)\le\frac{\alpha_1 d(T_x,F_y)d(T_x,T_y)}{d(T_x,T_y)+d(T_x,F_y)}+\alpha_2\left[d(T_x,E_x)+d(T_y,F_y)\right]+\alpha_3\left[d(T_x,F_y)+d(T_y,F_x)\right]+\alpha_4 d(T_x,T_y)$

Here we have generalized the contraction on five self maps as follows:

$\mathrm{T}_1,\mathrm{T}_2,\mathrm{T}_3,\mathrm{T}_4$ & T_5 on X to itself

[1.4] $d(T_1,T_2x,T_3,T_4y)\le C_1\left[\frac{d(T_5x,T_1T_2x)d(T_5y,T_3T_4y)}{d(T_5x,T_5y)}\right]+C_2\left[\frac{d(T_5x,T_3T_4y)d(T_5y,T_3T_4y)}{d(T_1T_2x,T_3T_4y)}\right]$

$+C_3\left[d(T_5x,T_1T_2x)+d(T_5y,T_3T_4y)\right]+C_4\left[d(T_5x,T_3T_4y)+d(T_5y,T_1T_2x)\right]$

$+C_5\left[d(T_5x,T_5y)+d(T_1T_2x,T_3T_4y)\right]+C_6 d(T_5x,T_5y).$

We see that under certain condition we get a unique common fixed point.

MAIN RESULTS

Let (X, d) be a complete metric space & $T_i : X \to X;\ \mathrm{i}=1,2,3,4,5$ be five mapping satisfying [1.4] and [2.1] as follows

[2.1] $\mathrm{T}_2 x\ne T_5 y;\quad \alpha_i\ge 0,$

$T_5(T_1T_2)=(T_1T_2)(T_5),\ T_5(T_3T_4)=(T_3T_4)(T_5)$

$T_1T_2=T_2T_1,\ T_3T_4=T_4T_3$

$T_1T_2(X)\subseteq T_5(X),\ T_3T_4(X)\subseteq T_5(X)$

Then $T_i(i=1\ to\ 5)$ have a unique common fixed point in X.

Proof: Let $x_0,x_1,x_2\in X$ and $T_1T_2(X)\subseteq T_5(X)$ such that

$T_1T_2(x_0)=T_5(x_1),\ T_3T_4(x_1)=T_5(x_2)$

$T_3T_4(X)\subseteq T_5(X)$

Since $T_1T_2x_{2n}=T_5x_{2n+1},\ T_3T_4x_{2n+1}=T_5x_{2n+1}$, replace $x\to x_{2n}$ & $y\to x_{2n-1}$, then we have from [1.4]

[2.3] $d(T_5x_{2n+1},T_5x_{2n})=d(T_1T_2x_{2n},T_3T_4x_{2n-1})$

$$d(T_1T_2x_{2n},T_3T_4x_{2n-1})\le C_1\frac{d(T_5x_{2n},T_1T_2x_{2n})d(T_5x_{2n-1},T_3T_4x_{2n-1})}{d(T_5x_{2n},T_5x_{2n-1})}$$

$$+C_2\frac{d(T_5x_{2n},T_3T_4x_{2n-1})d(T_5x_{2n-1},T_3T_4x_{2n-1})}{d(T_1T_2x_{2n},T_3T_4x_{2n-1})}$$

$$+C_3[d(T_5x_{2n},T_1T_2x_{2n})+d(T_5x_{2n-1},T_3T_4x_{2n-1})]$$

$$+C_4[d(T_5x_{2n},T_3T_4x_{2n-1})+d(T_5x_{2n-1},T_1T_2x_{2n})]$$

$$+C_5[d(T_5x_{2n},T_5x_{2n-1})+d(T_1T_2x_{2n},T_3T_4x_{2n-1})]$$

$$+C_6d(T_5x_{2n},T_5x_{2n-1}).$$

Which yields from (2.2) and using Property of Metric Space.

$$d(T_5x_{2n+1},T_5x_{2n})\{1-C_1-C_3-C_5\}\le d(T_5x_{2n-1},T_5x_{2n})(C_3+C_4+C_5)$$

Or

[2.4] $d(T_5x_{2n+1},T_5x_{2n})\le Kd(T_5x_{2n},T_5x_{2n-1})$

Where

$$0<K=\frac{(C_3+C_4+C_5)}{1-C_1-C_3-C_5}<1$$

Thus $[T_5x_{2n}]$ is a Cauchy Sequence in complete metric space X & have a common fixed point **u** in X such that

[2.5] $T_5x_{2n+1}=u=_{n\to\infty}T_5x_{2n}$

Now if $T_1T_2u\ne T_5u$ then

[2.6] $(T_1T_2u,T_5u)\le \lim d(T_1T_2u,T_3T_4T_5x_{2n+1})$

$$\le C_1\ \frac{\lim d(T_5u,T_1T_2u)d(T_5T_5x_{2n+1},T_3T_4T_5x_{2n+1})}{d(T_5u_1,T_5T_5x_{2n+1})}$$

$$+C_2\frac{d(T_5u,T_3T_4T_5x_{2n+1})d(T_5T_5x_{2n+1},T_3T_4T_5x_{2n+1})}{d(T_1T_2u,T_3T_4T_5x_{2n+1})}$$

$$+C_3[d(T_5u,T_1T_2u)+d(T_5T_5x_{2n+1},T_3T_4T_5x_{2n+1})]$$

$$+C_4[d(T_5u,T_3T_4T_5x_{2n+1})+d(T_5T_5x_{2n+1},T_1T_2u)]$$

$$+C_5[d(T_5u,T_5T_5x_{2n+1})d(T_1T_2u,T_3T_4T_5x_{2n+1})]+C_6[d(T_5u,T_5T_5x_{2n+1})]$$

Which Reduced to

$$d(T_1T_2u,T_5u)(1-C_5-C_4-C_3)\le 0.$$

[2.7] $d(T_1T_2u,T_5u)\le 0$

Which is a Contraction therefore

$T_1T_2(u) = T_5(u),$ Also we have $T_5(u) = T_3T_4(u),$ hence

[2.8] $T_1T_2(u) = T_5(u) = T_3T_4(u),$ and

[2.9] $T_5(T_5u) = T_5(T_1T_2u) = T_1T_2(T_1T_2u) = T_1T_2(T_3T_4u) = T_1T_2(T_5u) = T_5(T_3T_4u)$

$= T_3T_4(T_5u) = T_3T_4(T_1T_2u) = T_3T_4(T_3T_4u)$

Now if $T_1T_2u \neq T_3T_4(T_1T_2u)$

[2.10] $d(T_1T_2u, T_3T_4(T_1T_2u)) \leq C_1 \dfrac{d(T_5u, T_1T_2u)d(T_5T_1T_2u, T_3T_4T_3T_4u)}{d(T_5u, T_5T_3T_4u)}$

$+ C_2 \dfrac{d(T_5u, T_3T_4T_3T_4u)d(T_5T_3T_4u, T_3T_4T_3T_4u)}{d(T_1T_2u, T_3T_4T_3T_4u)}$

$+ C_3\left[d(T_5u, T_1T_2u) + d(T_5T_3T_4u, T_3T_4T_3T_4u)\right]$

$+ C_4\left[d(T_5u, T_3T_4T_3T_4u) + d(T_5T_3T_4u, T_1T_2u)\right]$

$+ C_5\left[d(T_5u, T_5T_3T_4u) + d(, T_1T_2u, T_3T_4T_3T_4u)\right] + C_6 d(T_5u, T_5T_3T_4u)$.

Therefore (2.10) reduces to

$d(T_5u, T_5T_5u)(1 - 2C_4 - 2C_5 - C_6) \leq 0$

Or equivalently

(2.11) $d(T_5u, T_5T_5u) \leq 0$

Which is a Contraction, therefore

$T_5(T_5(u)) = T_5(u)$

Also

$T_3T_4(T_1T_2u) = T_1T_2u$

Thus we get

Thus $T_1T_2u = v$ is a common fixed point of $T_5T_1T_2 \,\&\, T_3T_4$, if possible we take another common fixed point of $T_1T_2T_5 \,\&\, T_3T_4$.Thus

$\left[d(v, w)\right] = \left[d(T_1T_2v, T_3T_4w)\right]$

[2.12] which is a view of [1.4] yields

$$d(\nu,w)\le C_1\frac{[d(T_5\nu,T_1T_2\nu)]d(T_5w,T_5T_4w)}{d(T_5\nu,T_5w)}+C_2\frac{[d(T_5\nu,T_3T_4w)d(T_5w,T_3T_4w)]}{d(T_1T_2\nu,T_3T_4w)}$$
$$+C_3[d(T_5\nu,T_3T_4\nu)+d(T_5w,T_3T_2w)]+C_4[d(T_5\nu,T_3T_4w)+d(T_5w,T_1T_2\nu)]$$
$$+C_5[d(T_5\nu,T_5w)+d(T_1T_2\nu,T_3T_4w)]+C_6d(T_5\nu,T_5w)$$

Equation [2.12] reduced to

$$d(\nu,w)(1-2C_4-2C_5-C_6)\le 0$$

Or equivalently

$$d(\nu,w)=0\Rightarrow \nu=w$$

Now to show that ν is a unique common fixed point of T_1,T_2,T_3,T_4 & T_5 we find that

$$T_1(\nu)=T_1(T_1T_2\nu)=T_1(T_2T_1\nu)=T_1T_2(T_1\nu)\Rightarrow T_1\nu$$

Which implies that $T_1\nu$ is as another fixed point of T_1 & T_2 .Hence by uniqueness of fixed point of T_1 & T_2, we obtain ν a fixed point of T_1, similarly a unique fixed point of T_2,T_3,T_4 & T_5 .Hence proved.

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